Exam. Code : 211001

Subject Code : 4846

M.Sc. (Mathematics) 1st Semester

REAL ANALYSIS—I

Paper—MATH-551

Time Allowed—Three Hours] [Maximum Marks—100

Note :— (1) Attempt **FIVE** questions in all, taking at least **ONE** question from each part.

(2) Include the necessary results, depending upon the credit of the question.

(3) In the following, X denotes a metric space with metric d.

PART—I

1\. (a) Prove that finite intersection of open sets is open. Is the same true for infinite intersections ? 8

(b) Prove that countable union of countable sets is countable. 7

(c) Show that every subset of a discrete metric space is open as well as closed. 5

2\. (a) Prove that compact subsets of X are closed as well as bounded. 8

(b) Show that the set $\{z : |z| < 1\}$ is not a compact subset of the complex plane, by providing an open cover which has no finite subcover. 4

(c) Show that the Cantor set P is perfect. Give an infinite collection of perfect subsets of P. 8

PART—II

3. (a) Classify all connected subset of reals. 8

   (b) Let E be a finite connected subset of X. Prove that E is a singleton set. 4

   (c) Prove that every function of bounded variation is a sum as well as a difference of two monotone functions. 8

4. (a) Let $\{r_n\}$ be a sequence of all rational numbers. Prove that there exists a subsequence of $\{r_n\}$ convergent to π. 6

   (b) Show that every bounded sequence of reals has a convergent subsequence. 7

   (c) Let X be a complete metric space. Prove that X satisfies the Cantor intersection property. 7

PART—III

5. (a) Prove that countable intersection of dense open subsets of X is dense in X. 8

   (b) Prove that the set of all rational numbers is not a countable intersection of open subsets of reals. 5

   (c) State and prove the intermediate value theorem. 7

6. (a) Prove that there exists no monotone real function with uncountably many discontinuities. 10

(b) Prove that every uniformly continuous function maps Cauchy sequences onto Cauchy sequences. 10

PART—IV

7. (a) State and prove a Cauchy-Criterion for the Riemann integral. 10

(b) If f is Riemann integrable prove that so are | f | as well as f^2. Are the converses true ? 10

8. (a) Let f : [0, 1] $\rightarrow$ $\mathbb{R}$ be a function having only finitely many discontinuities. Prove that f is Riemann integrable. 10

(b) State and prove the first fundamental theorem of calculus for the Riemann integral. 10